Mechanics: Momentum and Impulse Worksheet Name: KEY

MomentumQuestions:

1. A 21 kg child on a 5.9 kg bike is riding with a velocity of 4.5 m/s to the northwest.

   a. What is the total momentum of the child and bike together?

   $m = 21\text{kg} + 5.9\text{kg} = 26.9\text{kg}$, $V = 4.5\text{m/s}$ $\quad p = mv = (26.9\text{kg})(4.5\text{m/s}) = \boxed{121.1\ \text{kg·m/s NW}}$

   b. What is the momentum of the child?

   $m = 21\text{kg}$, $V = 4.5\text{m/s}$ $\quad p = mv = (21\text{kg})(4.5\text{m/s}) = \boxed{94.5\ \text{kg·m/s NW}}$

   c. What is the momentum of the bike?

   $m = 5.9\text{kg}$, $V = 4.5\text{m/s}$ $\quad p = mv = (5.9\text{kg})(4.5\text{m/s}) = \boxed{26.6\ \text{kg·m/s NW}}$

2. What velocity must a 1210 kg car have in order to have the same momentum as a 2250 kg pickup truck traveling at 25 m/s to the east?

   $m_T = 2250\text{kg}$, $V_T = 25\text{m/s}$ $\quad p_T = m_T V_T = (2250\text{kg})(25\text{m/s}) = 56250\ \text{kg·m/s}$

   $p_c = 56250\ \text{kg·m/s}$, $m_c = 1210\text{kg}$, $V_c = x$ $\quad p_c = m_c V_c$

   $V_c = \frac{p_c}{m_c} = \frac{56250\ \text{kg·m/s}}{1210\text{kg}} = \boxed{46.5\text{m/s E}}$

Force and ImpulseQuestions:

3. A 0.50 kg football is thrown with a velocity of 15 m/s to the right. A stationary receiver catches the ball and brings it to rest in 0.020 s. What is the force exerted on the ball by the receiver?

   $F = x$, $t = 0.020\text{s}$, $m = 0.5\text{kg}$, $V = 15\text{m/s}$ $\quad F\Delta t = \Delta p \Rightarrow F = \frac{mv_f - mv_i}{\Delta t} = \frac{0 - (0.50\text{kg})(15\text{m/s})}{0.020\text{s}} = \boxed{375\text{N LEFT (-)}}$

4. A 0.40 kg soccer ball approaches a player horizontally with a velocity of 18 m/s to the north. The player strikes the ball and causes it to move in the opposite direction with a velocity of 22 m/s. What impulse was delivered to the ball by the player?

   $m = 0.4\text{kg}$, $V_f = -22\text{m/s}$, $V_i = 18\text{m/s}$ $\quad I = \Delta p = mv_f - mv_i = (0.4\text{kg})(-22\text{m/s}) - (0.4\text{kg})(18\text{m/s}) = \boxed{16\ \text{kg·m/s SOUTH (-)}}$

5. A 0.50 kg object is at rest. A 3.00 N force to the right acts on the object during a time interval of 1.50 s.

   a. What is the velocity of the object at the end of this interval?

   $m = 0.5\text{kg}$, $V_i = 0$, $V_f = x$, $F = 3\text{N}$, $t = 1.5\text{s}$ $\quad F\Delta t = \Delta p \Rightarrow F\Delta t = mv_f - mv_i \Rightarrow F\Delta t = mv_f - 0 \Rightarrow v_f = \frac{F\Delta t}{m} = \frac{(3\text{N})(1.5\text{s})}{0.5\text{kg}} = \boxed{9\text{m/s RIGHT}}$

   b. At the end of this interval, a constant force of 4.00 N to the left is applied for 3.00 s. What is the velocity at the end of 3.00 s?

   $m = 0.5\text{kg}$, $V_i = 9\text{m/s}$, $V_f = x$, $F = -4\text{N}$, $t = 3\text{s}$ $\quad F\Delta t = \Delta p \Rightarrow F\Delta t = mv_f - mv_i$

   $(-4\text{N})(3\text{s}) = (0.5\text{kg})v_f - (0.5\text{kg})(9\text{m/s})$

   $v_f = \boxed{-15\text{m/s LEFT}}$

Stopping Distance Questions:

6. A 2240 kg car traveling to the west slows down uniformly from 20.0 m/s to 5.00 m/s. How long does it take the car to decelerate if the force on the car is 8410 N to the east? How far does the car travel during the deceleration?

   $m = 2240\text{kg}$, $V_i = 20\text{m/s}$, $V_f = 5\text{m/s}$, $F = 8410\text{N}$ $\quad F\Delta t = \Delta p \Rightarrow F\Delta t = mv_f - mv_i$ $\quad \Delta t = \boxed{4\text{s}}$

   $(8410\text{N})\Delta t = (2240\text{kg})(5\text{m/s}) - (2240\text{kg})(20\text{m/s})$ $\quad a = \frac{\Delta v}{t} = \frac{5\text{m/s} - 20\text{m/s}}{4\text{s}} = -3.75$

   $v_f^2 = v_i^2 + 2ad \Rightarrow (5)^2 = (20)^2 + 2(-3.7$ $\quad d = \boxed{50\text{m}}$

7. How long would the car in question 6 take to come to a stop from its initial velocity of 20.0 m/s to the west? How far would the car move before stopping? SEE ABOVE

   $(-8410\text{N})\Delta t = (2240\text{kg})(0\text{m/s}) - (2240\text{kg})(20\text{m/s})$ $\quad \Delta t = 5.33\text{s}$

   $v_f^2 = v_i^2 + 2ad$ $\quad 0^2 = (20)^2 + 2(-3.75)d$ $\quad d = \boxed{53.3\text{m}}$

Conservation of Momentum Questions:

8. An 85.0 kg fisherman jumps from a dock into a 135.0 kg rowboat at rest on the west side of the dock. If the velocity of the fisherman is 4.30 m/s to the west as he leaves the dock, what is the final velocity of the fisherman and the boat?

   $m_1 = 85\text{kg}$, $m_2 = 135\text{kg}$, $V_1 = 4.3\text{m/s}$, $V_2 = 0$ $\quad m_1V_1 + m_2V_2 = (m_1 + m_2)V_f$

   $(85\text{kg})(4.3\text{m/s}) + (135\text{kg})(0) = (85\text{kg} + 135\text{kg})V_f$ $\quad V_f = \frac{365.5\ \text{kg·m/s}}{220\text{kg}} = \boxed{1.66\text{m/s W}}$

9. A boy on a 2.0 kg skateboard initially at rest tosses an 8.0 kg jug of water in the forward direction. If the jug has a speed of 3.0 m/s relative to the ground and the boy and the skateboard move in the opposite direction at 0.60 m/s, find the boy's mass.

   $m_s = 2\text{kg}$, $m_b = x$, $m_j = 8\text{kg}$, $V_j = 3\text{m/s}$, $V_s = -0.6\text{m/s}$ $\quad (m_s + m_b + m_j)V_i = (m_s + m_b)V_s + m_jV_j$

   $(2\text{kg} + x + 8\text{kg})0 = (2\text{kg} + x)(-0.6\text{m/s}) + (8\text{kg})(3\text{m/s})$

   $0 = -1.2\ \text{kg·m/s} - 0.6x\ \text{m/s} + 24\ \text{kg·m/s}$

   $x = \frac{22.8\ \text{kg·m/s}}{\quad} = \boxed{38\text{kg}}$

Perfectly Inelastic Collisions Questions:

10. A 1500 kg car traveling at 15.0 m/s to the south collides with a 4500 kg truck that is initially at rest at a stoplight. The car and the truck stick together and move together after the collision. What is the final velocity of the two-vehicle mass?

$m_c = 1500\,kg$ $m_t = 4500\,kg$ $m_cv_c + m_tv_t = (m_c+m_t)v_f$ $v_f = \frac{22500\,kg\cdot m/s}{6000\,kg} = 3.75$
$v_c = 15\,m/s$ $v_t = 0$ $(1500\,kg)(15\,m/s) + 0 = (1500\,kg + 4500\,kg)v_f$

11. A dry cleaner throws a 22 kg bag of laundry onto a stationary 9.0 kg cart. The cart and laundry bag begin moving at 3.0 m/s to the right. Find the velocity of the laundry bag before the collision.

$m_b = 22\,kg$ $v_b = x$ $v_f = 3\,m/s$ $m_bv_b + m_cv_c = (m_b+m_c)v_f$ $x = \frac{93\,kg\cdot m/s}{22\,kg} = 4.2\,m/s$ RIGHT
$m_c = 9\,kg$ $v_c = 0$ $(22\,kg)x + 0 = (22\,kg + 9\,kg)(3\,m/s)$

12. A 47.4 kg student runs down the sidewalk and jumps with a horizontal speed of 4.20 m/s onto a stationary skateboard. The student and the skateboard move down the sidewalk with a speed of 3.95 m/s. Find the following:

a. The mass of the skateboard. $m_sv_s + m_bv_b = (m_s+m_b)v_f$
$m_s = 47.4\,kg$ $v_s = 4.2\,m/s$ $(47.4\,kg)(4.2\,m/s) + 0 = (47.4\,kg + x)(3.95\,m/s)$
$m_b = x$ $v_b = 0$ $v_f = 3.95\,m/s$ $x = 3\,kg$

b. How fast the student would have to jump to have a final speed of 5.00 m/s.
$m_s = 47.4\,kg$ $v_s = x$ $m_sv_s + m_bv_b = (m_s+m_b)v_f$
$m_b = 3\,kg$ $v_b = 0$ $v_f = 5\,m/s$ $(47.4\,kg)x + 0 = (47.4\,kg + 3\,kg)(5\,m/s)$ $x = 5.32\,m/s$

Kinetic Energy in Perfectly Inelastic Collisions Questions:

13. A 0.25 kg arrow with a velocity of 12 m/s to the west strikes and pierces the center of a 6.8 kg target.

a. What is the final velocity of the combined mass?
$m_a = 0.25\,kg$ $v_a = 12\,m/s$ $m_av_a + m_tv_t = (m_a+m_t)v_f$
$m_t = 6.8\,kg$ $v_t = 0$ $v_f = x$ $(0.25\,kg)(12\,m/s) + 0 = (0.25\,kg + 6.8\,kg)x$ $\Rightarrow x = 0.43\,m/s\ W$

b. What is the decrease in the kinetic energy during the collision?
$KE_{a_i} = \frac{1}{2}(0.25\,kg)(12\,m/s)^2 = 18\,J$ $KE_f = \frac{1}{2}(0.25\,kg + 6.8\,kg)(0.43)^2 = 0.65\,J$ $\Delta KE = 18\,J - 0.65\,J = 17.35\,J$
$KE_{t_i} = 0$

14. During practice, a student kicks a 0.40 kg soccer ball with a velocity of 8.5 m/s to the south into a 0.15 kg bucket lying on its side. The bucket travels with the ball after the collision.

a. What is the final velocity of the combined mass?
$m_s = 0.4\,kg$ $v_s = 8.5\,m/s$ $m_sv_s + m_bv_b = (m_s+m_b)v_f$
$m_b = 0.15\,kg$ $v_b = 0$ $v_f = x$ $(0.4\,kg)(8.5\,m/s) + 0 = (0.4\,kg + 0.15\,kg)x$ $x = 6.18\,m/s\ S$

b. What is the decrease in the kinetic energy during the collision?
$KE_{s_i} = \frac{1}{2}(0.4\,kg)(8.5\,m/s)^2 = 14.45\,J$ $KE_f = \frac{1}{2}(0.4\,kg + 0.15\,kg)(6.18\,m/s)^2 = 10.50\,J$ $\Delta KE = 14.45\,J - 10.5\,J = 3.95\,J$
$KE_{b_i} = 0$

Elastic Collisions Questions:

15. A 16.0 kg (CANOE) moving to the left at 12.5 m/s makes an elastic head-on collision with a 14.0 kg raft moving to the right at 16.0 m/s. After the collision, the raft moves to the left at 14.4 m/s. Disregard and effects of the water.

a. Find the velocity of the canoe after the collision.
$= 16\,kg$ $m_R = 14\,kg$ $m_cv_{c_i} + m_rv_{r_i} = m_cv_{c_f} + m_rv_{r_f}$
$= 12.5\,m/s$ $v_{R_i} = -16\,m/s$ $(16\,kg)(12.5\,m/s) + (14\,kg)(-16\,m/s) = (16\,kg)x + (14\,kg)(14.4\,m/s)$ $x = -14.1\,m/s$ RIGHT
$= x$ $v_{R_f} = 14.4\,m/s$

b. Verify your answer by calculating the total kinetic energy before and after the collision.
$KE_{c_i} = \frac{1}{2}(16)(12.5)^2 = 1250\,J$ $\Sigma KE_i = 1250 + 1792 = 3042\,J$ $KE_{c_f} = \frac{1}{2}(16)(-14.1)^2 = 1590.5\,J$ $\Sigma KE_f = 1590.5 + 1451 = 3042\,J$
$KE_{r_i} = \frac{1}{2}(14)(-16)^2 = 1792\,J$ $KE_{r_f} = \frac{1}{2}(14)(14.4)^2 = 1451.5\,J$

16. A 25.0 kg bumper car moving to the right at 5.00 m/s overtakes and collides elastically with a 35.0 kg bumper car moving to the right. After the collision, the 25.0 kg bumper car slows to 1.50 m/s to the right, and the 35.0 kg bumper car moves at 4.50 m/s to the right.

a. Find the velocity of the 35.0 kg bumper car before the collision.
$= 25\,kg$ $m_2 = 35\,kg$ $m_1v_{1_i} + m_2v_{2_i} = m_1v_{1f} + m_2v_{2f}$
$= 5\,m/s$ $v_{2_i} = x$ $(25\,kg)(5\,m/s) + (35\,kg)x = (25\,kg)(1.5\,m/s) + (35\,kg)(4.5\,m/s)$ $x = 2\,m/s$ RIGHT
$= 1.5\,m/s$ $v_{2f} = 4.5\,m/s$

b. Verify your answer by calculating the total kinetic energy before and after the collision.
$KE_{1_i} = \frac{1}{2}(25)(5)^2 = 312.5\,J$ $KE_{1f} = \frac{1}{2}(25)(1.5)^2 = 28.125\,J$
$KE_{2_i} = \frac{1}{2}(35)(2)^2 = 70\,J$ $KE_{2f} = \frac{1}{2}(35)(4.5)^2 = 354.375\,J$
$382.5\,J$ $382.5\,J$